1. Four glasses are given. The first one contains a certain amount of apple juice, the second — peach juice, the third — grapefruit juice and the fourth — carrot juice. It is known that apple juice is sweeter (i.e., contains a higher concentration of sugar) than grapefruit juice, and peach juice is sweeter than carrot juice. Is it true that if we mix the contents of the first two glasses, the resulting mixture will be sweeter than the mixture of the contents of the third and the fourth glass?

2. The point $P = P(a; b)$ is located in the first quadrant. Consider a circle centered at the point P with radius greater than $\sqrt{a^2 + b^2}$, and denote the area of the part of this circle located in the i-th quadrant $(i = 1; 2; 3; 4)$ by S_i . Find $S_1 - S_2 + S_3 - S_4$.

3. Fuel efficiency $E(v)$ is measured in kilometers per liter (the number of kilometers covered by a vehicle travelling with constant velocity v per one liter of fuel). (The velocity v is measured in km/hour). Find the total amount of fuel used during a one hour drive, if $E(v) = \sqrt{v^3}/168$ and $v = 144\sqrt[4]{t}$.

4. The sequence x_n is defined by the recurrence relation

$$x_1 = x, \;\; x_{n+1} = \frac{(m-1)x_n + \dfrac{a}{x_n^{m-1}}}{m},$$

where a and x are positive real numbers, and m is a positive integer. Determine for which x, a and m the sequence converges and find its limit.

5. Workers were supposed to cover the floor of a rectangular room with tiles of two different sizes: 2×2 and 1×4 (the area of the floor is such that it can be covered completely by a certain combination of the two kinds of tiles). The necessary amount of tiles was obtained, but during their transportation to the destination three tiles of the size 2×2 were broken. It was decided to replace those three tiles with three 1×4 tiles. Prove that it is now impossible to cover the entire area of the floor with the available tiles. (Cutting tiles is not allowed.)

6. How many points with integer coordinates are there inside a sphere of radius 200 centered at the origin? Give an approximate answer with error no greater than 2%.

7. Let $P_n(x)$ be a polynomial of even degree n ($n > 1$), the coefficient of the highest degree of which is positive, and let $P_n(x) > P_n''(x)$ for every x. Prove that $P_n(x) > 0$ for every x.

8. Three players — Ben, Jen and Glen are seated around a pile of 2010 stones. Each player in their turn is allowed to remove one or two stones from the pile. Ben goes first, Jen — second, and Glen — third. According to the rules of the game, the player who takes the last stone wins 10$, the player whose turn is next receives 1$, and the player who made the previous move receives nothing. Assuming that Ben, Jen and Glen are all expert players, and they each make the optimal moves in order to win the greatest amount of money, knowing that the other players are trying to do the same, what will the result of this game be?

9. The rows of a determinant of a 3x3 matrix consist of three consecutive digits of certain three-digit numbers, all of which are divisible by 17. Prove that the determinant is also divisible by 17.

10. Is it possible to draw 8 parallel planes through the corners of a cube (each plane is to be drawn through a different corner of the cube) in such a way that the distances between every two adjacent planes will be equal?

11. It is known that any ant can move a breadcrumb across the surface of a table on its own. Several ants are trying to move a breadcrumb together, each one pushes as hard as it can, but they are directing their efforts in such a way that the crumb remains immobile (this means that the sum of the forces on it not necessarily equals to 0, but is not enough to overcome the force of friction between the crumb and the table.) Prove that there is an ant whose removal will make the crumb move.

12. The staff of the Sharikov Institute of Social Justice includes 15 researchers. The salary of each of the researchers equals a whole number of dollars and does not exceed 10$. (Zero salary is possible). Each month the laboratory director chooses 11 researchers and increases their salary by 1$. What is the minimal number of months the director needs in order to equalize all of the researchers' salaries given any initial distribution of salaries?